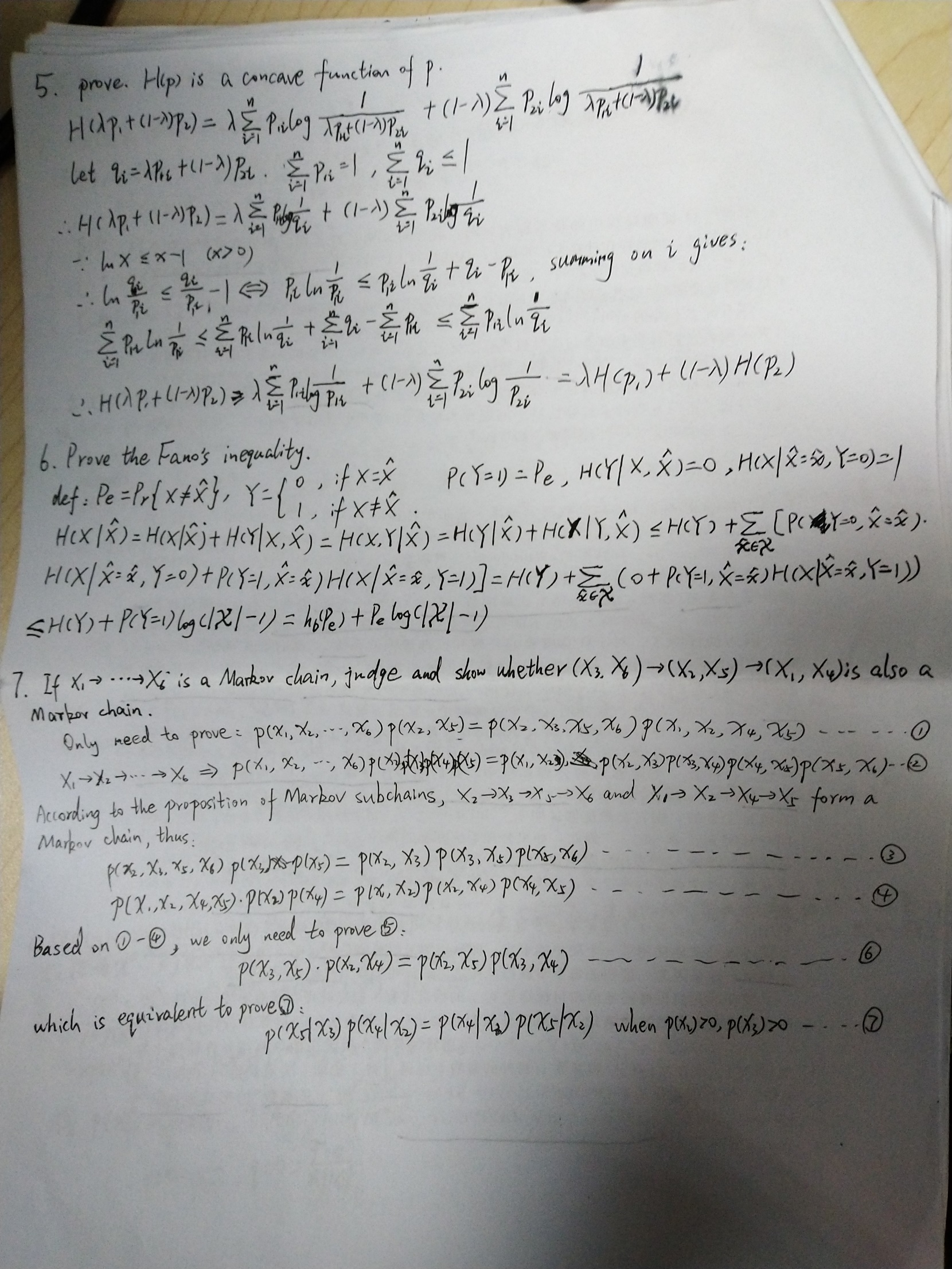
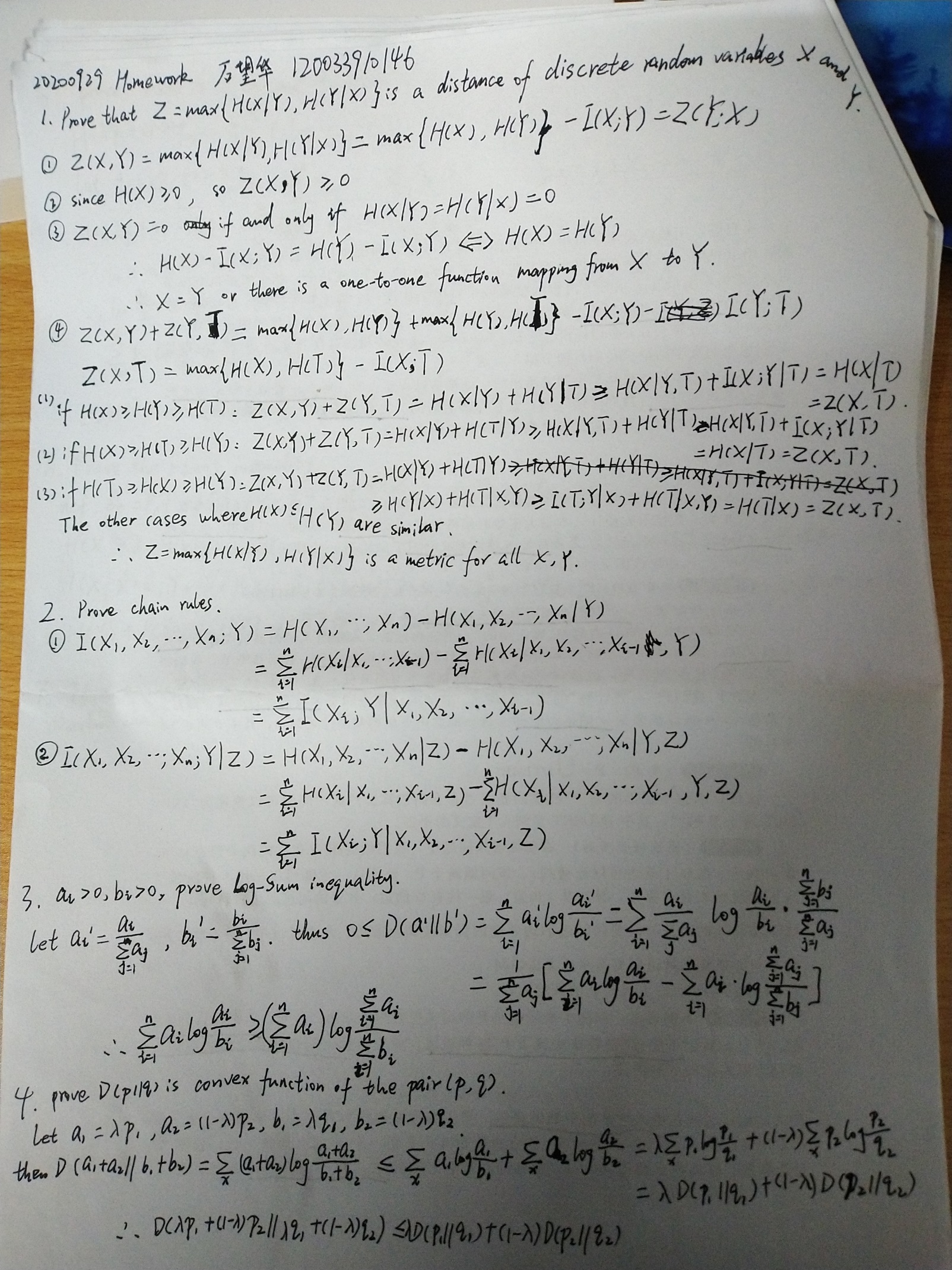
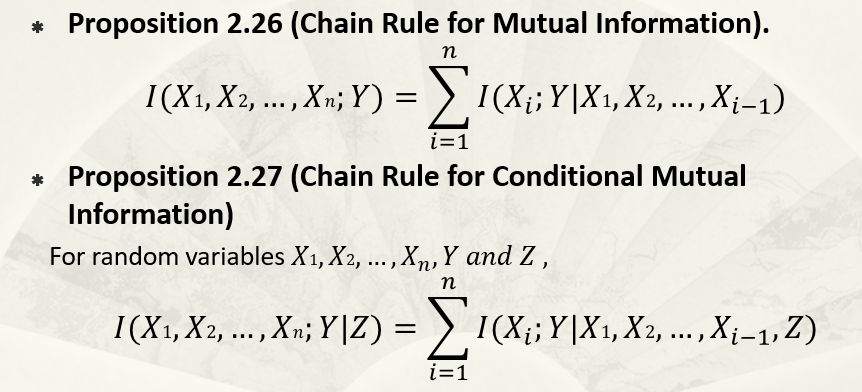
**20200929 Homework**



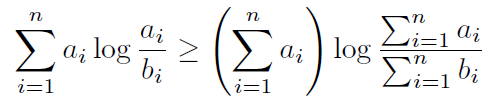
1. **Distance of discrete random variables.**

Prove that is a distances of discrete random variables *X* and *Y*.

**2. Prove the following two chain rules.**



**3. Log-Sum inequality**: For non-negative numbers and , prove

.

**4. Convex Relative Entropy**

If and are pairs of probability mass functions then



for all . That is, is **convex function** of the pair .

**5. Concave Entropy**

Let be the probability mass function of discrete random variable *X*. Here is denoted by . Prove that



That is, is a **concave function** of .

**6. Prove the Fano’s inequality.**

**7.** If is a Markov chain, **judge and show** whether is also a Markov chain.

**8. Remark. Convex and Concave mutual information**

Mutual Information can be expressed by a function of input distribution and transition distribution , i.e.,

.

1. For given input distribution , we say that is **convex** of transition distribution .
2. For given transition distribution, we say that is **concave** of input distribution .

**Convex 8A can be explained by:**

Let and be two joint distributions, we define

**Then, for given input distribution , is convex of transition distribution , i.e.,**

**i.e.**